

## **IMPLEMENTING SCHOOL-BASED ASSESSMENT: THE MATHEMATICAL THINKING ASSESSMENT (MATA) FRAMEWORK**

By

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### **1.0 Introduction**

The nature of assessment plays an important role in mathematics education. Teachers use assessment to help students to achieve the aims of mathematics curriculum by comprehensively accounting their learning over a period of time. As such, assessment of students' learning should not be interpreted as the end point of students' learning experiences; instead it serves as a mean to attain educational goals (Webb, 1993). Assessment, as defined by National Council of Teachers of Mathematics (NCTM) (1995), is "the process of gathering evidence about student's knowledge of, ability to use, and disposition toward, mathematics and making inferences from the evidence for a variety purposes" (p. 6). Hence, to promote effective classroom assessment, teachers are expected to involve actively in the four phases of assessment process: (a) planning, (b) gathering evidence, (c) interpreting evidence, and (d) using the results for decision making (NCTM, 1995) as illustrated in Figure 1.

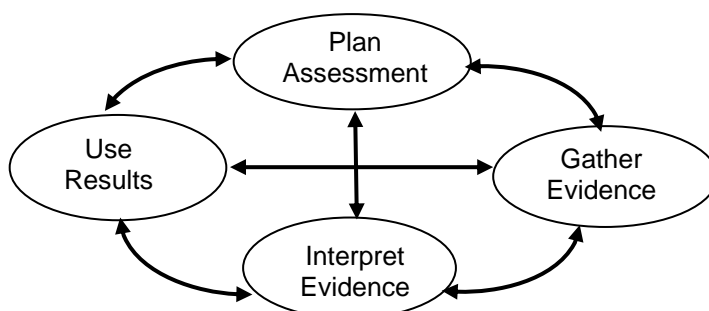


Figure 1: Four Phases of Assessment

These four phases of assessment process are interrelated. However, they are not to be carried out in linear sequential manner. For example, teachers need to use the information concerning students' current learning progresses before planning a test, or using the result obtained to determine what is the best way to gather and interpret the evidences in order to

improve the quality of the coming test. Once the assessment is properly plan, it will function as “an important tool for understanding the knowledge that students are constructing, the meaning that they are assigning to mathematical ideas, and the progress that they are making toward achieving mathematical power” (Webb, 1993, p.2). However, most of the teachers failed to make use of such evidences to plan and make sound instructional decisions because of “unacceptable low levels of assessment literacy among practicing teachers and administrators in our schools...” (Stiggins, 2001, p. 5). Due to this reason, assessment remains crucial in education, both in content and instructional approach (Neill et al, 1995).

## **2.0 School-based Assessment**

The Blueprint of Education Development (Ministry of Education Malaysia, 2007) pronounces that the greatest challenge faced by the Ministry of Education (MOE) in this decade is to lessen the examination oriented type of learning in the school. In order to achieve this goal, the blueprint suggested that school assessment be reformed by introducing a school-based grading system that emphasizes on task-based assessment. The aim of school-based assessment is to improve the quality of teaching, learning and assessment. Under this assessment format, teachers will be given greater responsibility in developing assessment and linking it to effective learning. Students’ achievements will be judge and graded based on the criteria and standards specified in subject syllabuses, and are moderated by review panels consisting of subject matter experts (Queensland Studies Authority, 2007).

However, implementing school-based assessment is not an easy task. Adi Badiozaman Tuah (2006) pointed out that there are three contributing factors: (a) the schools fail to interpret and comprehend assessment into wider operational terms that bring improvement to the learning and instruction in school; (b) the schools will forego the short-term instructional responsibilities, such as school-based assessment, in order to fulfill the interest of the public in getting good results in the public examinations; and (c) there is the human factor where teachers are not preparing or equipping themselves with the knowledge or skills that make school-based assessment as an integral part of the school-based curriculum development process.

Based on the remark made by Adi Badiozaman Tuah (2006), it is clear that the teachers are far from ready to implement the school-based assessment into our education system. They do not fully understand the concept of school-based assessment and lack the know-how in developing the assessment tasks. This lacking has led to another issue, the validity of the school-based assessment. According to McMillan (2001), “validity is a characteristic that refers to the appropriateness of the inferences, uses, and consequences that result from the assessment” (p. 59). Hence, the meaning

of validity in assessment is not confined to the extent a test measures what suppose to be measure, rather, how reasonable and acceptable the interpretation of the information collected through the assessment (McMillan).

Up to date, there is yet available assessment frameworks for our teachers to be used as a guide in executing school-based assessment. Hence, there is an urgent need to develop an assessment framework that is usable, valid and reliable, which may contribute to the success of assessment reform as stated in the Blueprint of Educational Development. In view of this urgency, the new assessment framework, the Mathematical Thinking Assessment (MaTA) framework is evolved.

### **3.0 Definition of Mathematical Thinking**

What is mathematical thinking? According to Lutfiyya (1998) and Cai (2002), there is yet to find a well defined meaning or explanation of mathematical thinking. To make the situation worse, the educators from different countries seem to define differently the meaning of mathematical thinking with respect to their mathematics curriculum. Hence, a well defined meaning of mathematical thinking must be established first before any study or research related to mathematical thinking can be conducted.

The word “mathematical thinking” is not used or stated explicitly in the Malaysian primary and secondary levels mathematics curriculum. However, a related statement, “to think mathematically” was used in the write-up of the main aim of secondary school mathematics curriculum.

The Mathematics curriculum for secondary school aims to develop individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decision. (MOE Malaysia, 2005, p.2)

The above statement denotes that mathematical thinking should be promoted in the Malaysian mathematics classroom if we are to produce future students who can think mathematically. Nonetheless, a closer analysis of the intended aim of secondary school mathematics curriculum shows that there are three components which constitute to the construction of mathematical thinking framework: content knowledge (mathematical knowledge); attitudes or disposition (effectively and responsibly); and mental operations (problem solving and decision making). These three components are found able to fit and incorporate into both the primary and the secondary school mathematics curriculum documents.

In view of the above discussion, mathematical thinking should include the following characteristics.

- It involves the manipulation of mental skills and strategies.
- It is highly influenced by the tendencies, beliefs or attitudes of a thinker.
- It shows the awareness and control of one's thinking such as meta-cognition.
- It is a knowledge-dependent activities (Lim & Hwa, 2006).

Base on these characteristics, this study defined mathematical thinking as mental operations which are supported by mathematical knowledge and certain kind of dispositions toward the attainment of solution to mathematics problem. Mathematical thinking is important particularly in the process of acquiring mathematical concepts and skills. However, teachers in schools are not aware of the importance of thinking in Mathematics and hence do not emphasize it in the development of students' intellectual growth (MOE, 1993). Thus, many students fail to engage thinking skills in solving complex real life problems. In the words of Von Glaserfeld (1995):

[Educators] have noticed that many students were quite able to learn the necessary formula and apply them to the limited range of textbook and test situation, but when faced with novel problem, they fell short and showed that they were far from having understood the relevant concepts and conceptual relations. (p. 20)

Therefore, an effective assessment framework is needed to promote students' mastery of mathematical thinking through the classroom learning. Without appropriate assessment and grading system in assessing mathematical thinking, we cannot know how effective and efficient a teacher is at teaching mathematical thinking or how skilful a student is at mathematical thinking. Nor can we know what needs to be attended to in order to promote the teaching and learning of mathematical thinking in the classroom.

#### **4.0 The Mathematical Thinking Assessment (MaTA) Framework**

The Mathematical Thinking Assessment (MaTA) framework consists of four components: (a) performance assessment, (b) Metacognition Rating Scale, (c) Mathematical Dispositions Rating Scale, and (d) Mathematical Thinking Scoring Rubric. The MaTA will be implemented by teachers in the school context to assess students' mathematical thinking: the performance assessment will be administered to elicit students' thinking process while solving the mathematical problem; the Metacognition Rating Scale will be used to specify students' awareness, such as, monitoring and reflection, during the problem solving process; the Mathematical Dispositions Rating Scale will be used to indicate students' predisposition toward learning of mathematics; whereas the Mathematical Thinking Scoring Rubric will be used to score and grade students' mathematical thinking according to the

domains defined in this study. The conceptual framework of MaTA is illustrated in Figure 2. Detail description of how this framework could be implemented in the school context is given in the following sections.

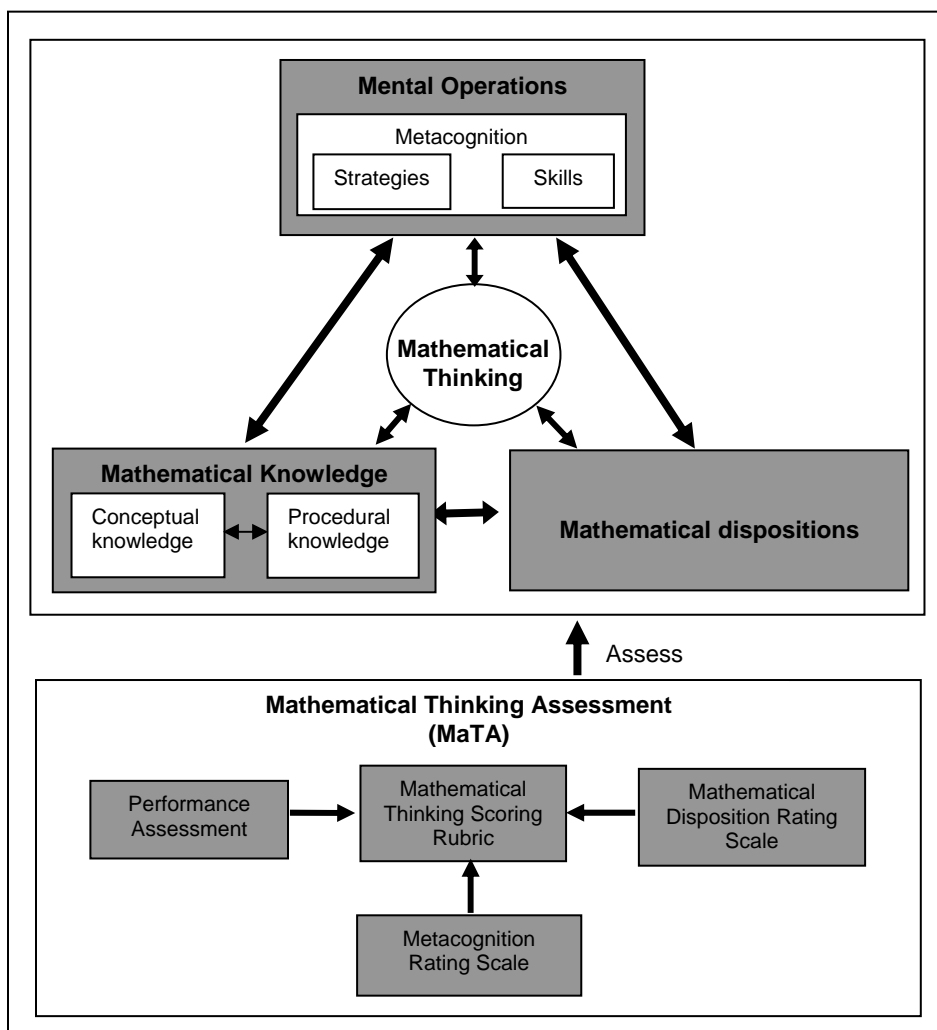


Figure 2: Conceptual Framework of MaTA Framework

#### 4.1 Performance Assessment

Performance assessment is a type of school-based assessment which allows the students to demonstrate their skills and knowledge in real life context. Through the demonstration of problem solving strategies, students' mathematical thinking could be revealed. Hence, it is very important to design and select the performance tasks that are able to elicit students' mathematical thinking. The performance tasks which are carefully designed and selected will determine the success of implementing performance

assessment in the school context. Figure 3 illustrates how to plan a valid and reliable performance assessment that could be used to assess students' mathematical thinking.

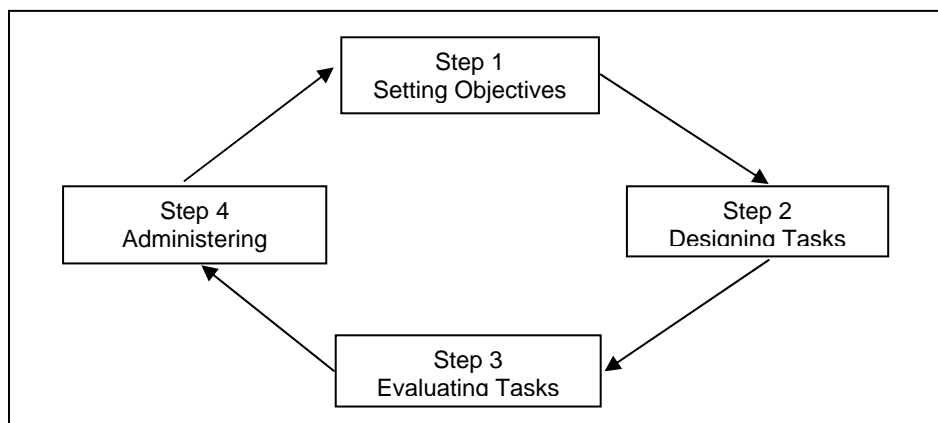


Figure 3: Planning Performance Assessment

### **Step 1: Setting objectives for performance assessment**

When planning performance assessment, it is important to set the objectives of the assessment. By setting the objectives, the teachers will be able to know exactly what learning outcomes they anticipate from their students. Furthermore, these objectives will guide the teachers to select valid and reliable tasks that meet the expectation and the objectives of the assessment.

### **Step 2: Designing performance tasks**

Performance tasks should be designed with open-ended format which allow alternative interpretations or solutions that ask for explanations and reasoning. Hence, it is important to start designing the performance tasks by referring to questions or problems that are well established, such as, from textbooks, reference books, internet resources or assessment institutions such as TIMSS, NAEP and PISA. Teachers could adopt the problem and modify it so that it will suit the Malaysian Mathematics Curriculum and the objectives of the assessment set by the teachers.

The good and effective performance tasks exhibit the following characteristics.

- (a) The tasks are open-ended in nature.
- (b) The tasks are authentic and real-life-based.
- (c) The tasks can be solved by using multiple approaches or solutions.
- (d) The tasks adequately represent the skills and knowledge you expect students to attain.
- (e) The tasks are structured to provide measures of several goals or objectives of the assessment.

- (f) The tasks must match specific instructional intentions, such as the learning objectives that are specified in each of the mathematics topic.

Therefore, it is very important for teachers to examine the designed task carefully so that it meets all the criteria mentioned earlier. This is to ensure that the task is challenging and is able to elicit students' mathematical thinking while they try to solve the given task.

**Step 3: Evaluating performance tasks**

Once the tasks are designed, the teacher can engage the following steps (Table 1) to investigate and evaluate the suitability of the tasks that will meet the objectives of the assessment.

Table 1: Checklist for Evaluation Performance Tasks

Item	Description	Check
1.	Perform the task(s).	
2.	The solution(s) is reasonable and it meets the requirement of the mathematics syllabus.	
3.	List the important aspects of the performance which are related to the objectives of the assessment.	
4.	Make sure the performance criteria can be expressed in terms of observable student behaviors or product characteristics.	
5.	Make sure the performance criteria are arranged in the order in which they are likely to be observed.	
6.	Seek second opinion to improve the performance tasks; it can be done by asking other teachers to do it or piloted it to a few students.	

**Step 4: Administering performance assessment**

Before the performance assessment is being administered, make sure that the students are aware of the evaluation criteria specified in the Mathematical Thinking Scoring Rubric (Table 2. See Appendix A). This can be done by:

- (a) providing the Mathematical Thinking Scoring Rubric to each of the students;
- (b) discussing with the students each of the performance criteria and the levels of performance specified in this scoring rubric;
- (c) discussing with the students how their mathematics written solutions are being assessed through this scoring rubric;
- (d) discussing with the students different types of approach that could be used in attempting the same task in the performance assessment; and
- (e) more importantly, constantly promoting performance assessment during teaching and learning in the classroom by giving them real life problems to solve; asking them to reason and verify their solutions; and remind them whether they have achieved the satisfactory levels of performance in the Mathematical Thinking Scoring Rubric.

#### **4.2 Metacognition Rating Scale**

Metacognition, as defined by Beyer (1988), “consists of those operations by which we direct and control these meaning making strategies and skills. ... Any act of thinking involves a combination of operations designed to produce meaning and to direct how that meaning is produced” (p. 47). He further claimed that metacognition is also associated closely to the knowledge, cognitive operation and dispositions that constitute to the thinking activities. In this assessment framework, the scoring criteria for metacognition developed by MacLeod, Butler and Syer (1996) is adopted and used as the measurement of skill levels in Metacognition Rating Scale (Figure 4. See Appendix B).

According to MacLeod, Butler and Syer (1996), the measures of metacognition can be divided into three categories. These categories are: students’ understanding about tasks, students’ understanding about strategies, and students’ management of the learning process. Based on these categories, six items of Metacognition Rating Scale is developed. Teachers will determine the scale level of Metacognition Rating Scale for each individual student based on their observation in the classroom right from the beginning of the year. The metacognition scale level will be documented each time after the students taking the performance assessment. Firstly, the teachers will rate item by item students’ metacognition through the Metacognition Rating Scale. After that, the teachers will determine an overall band by referring to the scoring guide given, such as, if the student’s total rating score is 21; he/she will be awarded Band 4 and this overall band will represent student’s level of metacognition in the Mathematical Thinking Scoring Rubric (Table 2. See Appendix A).

#### **4.3 Mathematical Disposition Rating Scale**

In order to learn mathematics, one has to know the mathematical knowledge and skills that constitute the subject. However, knowing the concepts, procedural and application does not promise one to be proficient and successful in mathematics problem solving. There are other domains, such as dispositions toward mathematics that need to be emphasized in the learning process. Students need to show how they appreciate mathematics if they want to become a skillful mathematics problem solver. According to NCTM (1989), mathematical dispositions refer:

...not simply to attitudes but to a tendency to think and to act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks - whether with confidence, willingness to explore alternatives, perseverance, and interest - and in their tendency to reflect on their own thinking. (p. 233)



Therefore, having the positive dispositions toward mathematics is equally important as mathematical knowledge and skills. These dispositions will help the students to organize knowledge and skills into coherent function, and hence produce effective and skillful thinking in the attempt to solve complex mathematics problem in real life situation. The information concerning students' mathematical dispositions "is best collected through informal observation of students as they participate in class discussions, attempt to solve problems, and work on various assignments individually or in groups (NCTM, 1989, p. 233). In this framework, the Mathematical Disposition Rating Scale (Figure 5. See Appendix C) is developed based on the criteria of mathematical dispositions proposed by NCTM (1995), with the aim to capture students' dispositions toward the learning of mathematics in the classroom.

The students' mathematical dispositions rating process started at the beginning of the year until the students completed the performance assessment. Teachers will have to constantly observe students' dispositions while they participate in classroom activities, such as discussion, solving mathematics problem, and working on various assignments individually or in groups. Firstly, the teachers will have to rate item by item students' dispositions in the Mathematical Disposition Rating Scale. After that, the teachers will have to determine an overall band by referring to the scoring guide stated in the Mathematical Disposition Rating Scale. For example, if the student's total rating score is 13; he/she will be awarded Band 2 and this overall band will represent the student's level of mathematical dispositions in the Mathematical Thinking Scoring Rubric (Table 2. See Appendix A).

#### **4.4 Mathematical Thinking Scoring Rubric**

Mathematical Thinking Scoring Rubric is a scoring system where it guides the teachers to assess students' mathematics achievement "qualitatively". Contrary to traditional assessment, scoring rubric is a descriptive scoring scheme that provide informatics feedback not only on the products of students' learning, but also the processes of students' learning (Brookhart, 1999). Tierney and Simon (2004) further argued that scoring rubric contains qualitative descriptions of the performance criteria. Therefore, it is especially useful in assessing learning within the process of formative evaluation in the classroom. In this study, the Mathematical Thinking Scoring Rubrics is developed and used to assess students' mathematical thinking, in terms of their mathematical knowledge (conceptual and procedural), mental operations (strategies, skills and metacognition) and mathematical dispositions.

In order to produce a valid and reliable scoring system, teachers have to understand and familiarise with the judging criteria for each of the domains in Mathematical Thinking Scoring Rubric. This scoring system will help teachers to focus on the elements or steps in the students' written solutions,

and subsequently give the most accurate scores to the students with respect to each of the domains. Table 2 (See Appendix A) shows the complete set of Mathematical Thinking Scoring Rubric that could be used by teachers to assess students' levels of performance for each of the domain of mathematical thinking during mathematics problem-solving.

Once the teachers are familiar with the scoring criteria for each of the domain in Mathematical Thinking Scoring Rubric, teachers could begin the scoring process by following the steps highlighted below.

**Step 1: Collect students' Mathematics written performance**

After administering the performance assessment, collect all the students' written solutions. Make sure that the students use appropriate approaches to perform the tasks, as required by the MaTA framework.

**Step 2: Scoring students' performance**

- (i) By referring to the scoring criteria for each of the domains in Mathematical Thinking Scoring Rubric, namely conceptual knowledge, procedural knowledge, thinking strategies and thinking skills, teachers could score their students' levels of performance respectively based on their written solutions.
- (ii) As for metacognition domain, the overall scale given by the teachers in Metacognition Rating Scale will represent students' levels of performance in Mathematical Thinking Scoring Rubric.
- (iii) Similarly, the levels of performance for students' mathematical dispositions will be determined by the teachers through Mathematical Depositions Rating Scale.

**Step 3: Reporting students' Mathematics performance**

After scoring students' written solutions based on Mathematical Thinking Scoring Rubric, students' levels of performance for each domain could be summarized into a standard report, Teacher's Report on Student's Mathematical Thinking (Figure 6. See Appendix D). The teacher's comments for each domain of mathematical thinking will be focused on to what extent the students are performing, based on their written solutions and teachers' classroom observation. Parents' feedback column is included in this report because two ways communication is also one of the factors that could improve students' mathematics performance.

## **5.0 Conclusion**

Even though our mathematics curriculum emphasises the formation of mathematical concepts and skills through problem solving, communication, reasoning and connecting (MOE, 2005), it does not really reflect on the standard-based assessment implemented in the school. This is because the traditional standard-based assessment often restricted to the correctness of final answers and this resulted in teachers and students paying more attention to the 'know what' knowledge rather than 'know how' knowledge. Hence, the Mathematical Thinking Assessment (MaTA) framework provides an alternative way of assessment approach for teachers to examine their

students' thinking processes, or mathematical thinking. Teachers could identify students' different cognitive and affective constructs of mathematical thinking through MaTA, and the information gathered can be served as a tool to diagnose students' areas of learning difficulty. With MaTA feedback, teachers could have designed specific lessons to help and enhance students' mathematical understanding; and more importantly, students are able to use their knowledge and skills in solving real life problems.

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Table 2: Mathematical Thinking Scoring Rubric

Levels of Performance	Mathematical Knowledge			Mental Operations			Mathematical Depositions	
	Conceptual Knowledge	Procedural Knowledge	Thinking Strategies	Thinking Skills	Metacognition			
Band 5 (Excellent Solver)	<ul style="list-style-type: none"> <li>Complete understanding of concept(s); able to recognize and interpret all the concept(s) by using models, diagram or symbol</li> <li>Apply the concept(s) to solve problem correctly</li> </ul>	<ul style="list-style-type: none"> <li>Completely recognize and select an appropriate procedure</li> <li>Execute all the procedure reliably and efficiently</li> <li>Give complete reason for each of the steps in the procedure</li> </ul>	<ul style="list-style-type: none"> <li>Complete posing and formulating of mathematical problem</li> <li>Select an efficient &amp; appropriate method or strategy</li> <li>Judge the correctness of solution perfectly</li> </ul>	<ul style="list-style-type: none"> <li>Link all the mathematical idea(s) to other learning area or real life situation</li> <li>Using all the correct terms and notations to express mathematical idea(s)</li> <li>Show perfect logical/mathematical sense towards the answer obtained</li> </ul>	The student always (80% - 100%)	<ul style="list-style-type: none"> <li>has a clear overview of the mathematical task and the link between the information in the task.</li> <li>judge the quality of his/her own task performance.</li> <li>has a clear and specific description about strategies used to approach the mathematical task.</li> </ul>	The student always (80% - 100%)	<ul style="list-style-type: none"> <li>confident in using mathematics to solve problems, to communicate ideas, and to reason.</li> <li>identify errors in the answers, and in the use of appropriate mathematical symbols or logic.</li> </ul>
Band 4 (Good Solver)	<ul style="list-style-type: none"> <li>Almost complete understanding of concept(s); able to recognize and interpret most of the concept(s) by using models, diagram or symbol</li> <li>Apply the concept(s) to solve problem with minor error</li> </ul>	<ul style="list-style-type: none"> <li>Almost completely recognize and select an appropriate procedure</li> <li>Execute most of the procedure reliably and efficiently</li> <li>Give incomplete reason for each of the steps in the procedure</li> </ul>	<ul style="list-style-type: none"> <li>Nearly complete posing and formulating of mathematical problem</li> <li>Select a nearly efficient &amp; appropriate method or strategy</li> <li>Judge the correctness of solution incompletely</li> </ul>	<ul style="list-style-type: none"> <li>Link almost all the mathematical idea(s) to other learning area or real life situation</li> <li>Using almost correct terms and notations to express mathematical idea(s)</li> <li>Show incomplete logical/mathematical sense towards the answer obtained</li> </ul>	The student often (60% - 79%)	<ul style="list-style-type: none"> <li>link between strategy elements and his/her perception of the task.</li> <li>Provides a clear assessment of problem or show awareness of the process of learning.</li> <li>control the thoughts or emotions that help him/her to stay on the mathematical task.</li> </ul>	The student often (60% - 79%)	<ul style="list-style-type: none"> <li>persistent in his/her solution, never give up.</li> <li>find ways to get unstuck in a problem</li> <li>apply mathematical knowledge and skills in other subjects matter and also in daily life.</li> </ul>
Band 3 (Modest Solver)	<ul style="list-style-type: none"> <li>Partial understanding of concept(s); partially recognize and interpret the concept(s) by using models, diagram or symbol</li> <li>Apply the concept(s) to solve problem with major error</li> </ul>	<ul style="list-style-type: none"> <li>Partially recognize and select an appropriate procedure</li> <li>Execute part of the procedure reliably and efficiently</li> <li>Give no reason for each of the steps in the procedure</li> </ul>	<ul style="list-style-type: none"> <li>Partial posing and formulating of mathematical problem</li> <li>Select a partially efficient &amp; appropriate method or strategy</li> <li>No judgment on the correctness of solution</li> </ul>	<ul style="list-style-type: none"> <li>Partially link the mathematical idea(s) to other learning area or real life situation</li> <li>Using partially correct terms and notations to express mathematical idea(s)</li> <li>Unable to show logical/mathematical sense towards the answer obtained</li> </ul>	The student sometimes (40% - 59%)	<ul style="list-style-type: none"> <li>link between strategy elements and his/her perception of the task.</li> <li>Provides a clear assessment of problem or show awareness of the process of learning.</li> <li>control the thoughts or emotions that help him/her to stay on the mathematical task.</li> </ul>	The student sometimes (40% - 59%)	<ul style="list-style-type: none"> <li>interested to explore various methods in order to obtain the easier or simpler solution to the mathematical problem.</li> </ul>
Band 2 (Limited Solver)	<ul style="list-style-type: none"> <li>No understanding of concept(s); unable to recognize and interpret the concept(s) by using models, diagram or symbol</li> <li>Try but unable to apply the correct concept(s) to solve the problem</li> </ul>	<ul style="list-style-type: none"> <li>Unable to recognize and select an appropriate procedure</li> <li>Unable to execute procedure reliably and efficiently</li> <li>Give no reason for each of the steps in the procedure</li> </ul>	<ul style="list-style-type: none"> <li>Unable to pose and formulate the mathematical problem</li> <li>Select an inefficient &amp; inappropriate method or strategy</li> <li>No judgment on the correctness of solution</li> </ul>	<ul style="list-style-type: none"> <li>Unable to connect the mathematical idea(s) to other learning area or real life situation</li> <li>Unable to use correct terms and notations to express mathematical idea(s)</li> <li>Unable to show logical/mathematical sense towards the answer obtained</li> </ul>	The student seldom (20% - 39%)	<ul style="list-style-type: none"> <li>control the thoughts or emotions that help him/her to stay on the mathematical task.</li> </ul>	The student seldom (20% - 39%)	<ul style="list-style-type: none"> <li>appreciates the importance of mathematics in our culture and its value as a tool and as a language</li> </ul>
Band 1 (Non Solver)	<ul style="list-style-type: none"> <li>Blank</li> <li>Copy information only</li> <li>No sign of trying</li> </ul>	<ul style="list-style-type: none"> <li>Blank</li> <li>Copy information only</li> <li>No sign of trying</li> </ul>	<ul style="list-style-type: none"> <li>Blank</li> <li>Copy information only</li> <li>No sign of trying</li> </ul>	<ul style="list-style-type: none"> <li>Blank</li> <li>Copy information only</li> <li>No sign of trying</li> </ul>	The student never (0% - 19%)		The student never (0% - 19%)	

## Appendix B

<b>Mathematical Thinking Assessment (MaTA) Framework</b>																							
<b>Metacognition Rating Scale</b>																							
Student's Name	: Jamalludin bin Ahmad	Overall Band	<b>4</b>																				
Class	: 4 Sains 2																						
<p>Use the following scale to mark your student in each criterion.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 10px;">5</td> <td style="padding-right: 10px;">=</td> <td style="padding-right: 10px;">Always</td> <td style="padding-right: 10px;">(80% - 100%)</td> </tr> <tr> <td>4</td> <td>=</td> <td>Often</td> <td>(60% - 79%)</td> </tr> <tr> <td>3</td> <td>=</td> <td>Sometime</td> <td>(40% - 59%)</td> </tr> <tr> <td>2</td> <td>=</td> <td>Seldom</td> <td>(20% - 39%)</td> </tr> <tr> <td>1</td> <td>=</td> <td>Never</td> <td>(0% - 19%)</td> </tr> </table>				5	=	Always	(80% - 100%)	4	=	Often	(60% - 79%)	3	=	Sometime	(40% - 59%)	2	=	Seldom	(20% - 39%)	1	=	Never	(0% - 19%)
5	=	Always	(80% - 100%)																				
4	=	Often	(60% - 79%)																				
3	=	Sometime	(40% - 59%)																				
2	=	Seldom	(20% - 39%)																				
1	=	Never	(0% - 19%)																				
			What I give to student																				
<b>Student's Understanding About Tasks</b>																							
1	He/she has a clear overview of the mathematical task and able to link the information in the task to one another.		4																				
2	He/she provides a clear description of how to judge the quality of his/her own task performance.		2																				
<b>Student's Understanding About Strategies</b>																							
3	He/she gives a clear and specific description about the strategies used to approach the mathematical task.		4																				
4	The link between strategy elements and his/her perception of the task is interconnected and appropriate.		3																				
<b>Student's Management of the learning Process</b>																							
5	He/she provides a clear assessment of problem or an awareness of the process of learning.		5																				
6	He/she is able to control the thoughts or emotions that help him/her to stay on the mathematical task.		3																				
<b>Scoring Guidelines:</b>																							
		Total	21																				
<i>Band 1 (6 – 8)</i>	<i>Band 3 (15 – 20)</i>	<i>Band 5 (27 -30)</i>																					
<i>Band 2 (9 – 14)</i>	<i>Band 4 (21 – 26)</i>																						

Figure 4: Example of Teacher's Rating in Metacognition Rating Scale

## Appendix C

<b>Mathematical Thinking Assessment (MaTA) Framework</b>																						
<b>Mathematical Dispositions Rating Scale</b>																						
Student's Name	: Kho Tai Tong	Overall Band																				
Class	: 4 C	2																				
<div style="border: 3px double black; padding: 5px;"> <p>Use the following scale to mark your student in each criterion.</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td>5</td><td>=</td><td>Always</td><td>(80% - 100%)</td></tr> <tr><td>4</td><td>=</td><td>Often</td><td>(60% - 79%)</td></tr> <tr><td>3</td><td>=</td><td>Sometime</td><td>(40% - 59%)</td></tr> <tr><td>2</td><td>=</td><td>Seldom</td><td>(20% - 39%)</td></tr> <tr><td>1</td><td>=</td><td>Never</td><td>(0% - 19%)</td></tr> </table> </div>			5	=	Always	(80% - 100%)	4	=	Often	(60% - 79%)	3	=	Sometime	(40% - 59%)	2	=	Seldom	(20% - 39%)	1	=	Never	(0% - 19%)
5	=	Always	(80% - 100%)																			
4	=	Often	(60% - 79%)																			
3	=	Sometime	(40% - 59%)																			
2	=	Seldom	(20% - 39%)																			
1	=	Never	(0% - 19%)																			
		What I give to student																				
1.	He/She is confident in using mathematics to solve problems, to communicate ideas, and to reason.	2																				
2.	He/She is able to identify errors in the answers, and in the use of appropriate mathematical symbols or logic.	1																				
3.	When a strategy doesn't work, he/she tries another one instead of giving up.	2																				
4.	Whenever he/she gets stuck in the solution, he/she can always find ways to get unstuck.	1																				
5.	He/She is able to apply the mathematical knowledge and skills in other subjects matter and also in daily life.	1																				
6.	He/She is interested to explore various methods in order to obtain the easier or simpler solution to the mathematical problem.	3																				
7.	He/She appreciates the important of mathematics in our culture and its value as a tool and as a language	3																				
<b>Scoring Guidelines:</b>		<b>Total</b>																				
<i>Band 1 (7 – 10)</i>	<i>Band 3 (18 – 24)</i>	<i>Band 5 (32 -35)</i>																				
<i>Band 2 (11 – 17)</i>	<i>Band 4 (25 – 31)</i>	<b>13</b>																				

Figure 5: Example of Teacher's Rating in Mathematical Disposition Rating Scale

**Appendix D**


<b>Mathematical Thinking Assessment (MaTA) Framework</b>			
<b>Teacher's Report on Student's Mathematical Thinking</b>			
Name: Kamariah bt Hassan	Overall Band	<b>2.5</b>	
Class: 4A			
Domain	Band Level	Teacher's Comment	
Conceptual knowledge	4	<i>Able to use and apply most of the concepts correctly.</i>	
Procedural Knowledge	3	<i>Applies part of the mathematical procedures and solve the problems with several major errors and/or omissions</i>	
Thinking Strategies	2	<i>Unable to choose appropriate plans and strategies to solve the problem, and gave no justification to the solutions obtained.</i>	
Thinking Skills	3	<i>Link and communicates the mathematical ideas incompletely. Also, rarely uses appropriate mathematical terms and notations.</i>	
Metacognition	2	<i>Seldom monitor her problem solving process and hence unable to point out her mistake.</i>	
Mathematical Dispositions	2	<i>She shows little interest in mathematics problem solving. She needs a lot of encouragement and motivation to follow the mathematics lessons.</i>	
<b>Total Band Score</b>	<b>16</b>		
 _____ Teacher's Signature		23/7/2008 _____ Date	
<b>Parent's Feedback:</b>			
_____			
_____			
_____			
_____ Parent's Signature		_____ Date	
<b>Scoring Guide</b>			
<i>Band 0 (0 - 2)</i>	<i>Band 1.5 (8 - 10)</i>	<i>Band 2.5 (14 - 16)</i>	<i>Band 3.5 (20 - 22)</i>
<i>Band 1 (3 - 7)</i>	<i>Band 2 (11 - 13)</i>	<i>Band 3 (17 - 19)</i>	<i>Band 4 (23 - 24)</i>

Figure 6: Example of Teacher's Report on Student's Mathematical Thinking